Math 253 - Spring 2008 **Ch.14 & 15 Overview** **-Tsuchida**

Chapter 14 – Differentiating Functions of Many Variables

14.1 The Partial derivative
- Limit definitions of partial derivatives
- Visualizing partial derivatives on a graph (slope of tangent line in a direction)
- Estimating partial derivatives from tables and contour diagrams
- Using units to interpret partial derivatives in application problems

14.2 Computing partial derivatives algebraically
- Finding formulas for partial derivatives using techniques from Calculus I
  (This includes using the product rule, quotient rule and chain rule.)
- Interpretation of partial derivatives (vibrating string example)

14.3 Local linearity and the differential
- Tangent plane to a surface at a point
- Tangent plane approximations
- The differential

14.4 Gradients and directional derivatives (of functions of 2 variables)
- Constructing a unit-vector
- Calculating the gradient vector
- Geometric properties of the gradient vector
- Finding directional derivative by dotting unit-vector with gradient vector

14.5 Gradients and directional derivatives of functions of 3 variables
- Geometric properties of the gradient vector of functions of 3 variables

14.6 Chain rule
- Using tree diagrams to determine chain rule formulas in various situations
- Related rates problems (Parallel resistor problem, etc…)

14.7 Second-order partial derivatives
- Geometric interpretations: concavity in a given direction, and "twist" in a direction
- Determine signs of 2^{nd} order partial derivatives from a contour diagram
- Mixed partial theorem (You will <u>not</u> need to prove or justify this theorem)
- Taylor approximations

Chapter 15 Local Extrema

15.1 Local Extrema on an unbounded domain
- General procedure: (1) Identify and find formula for objective function f. (2) Identify and find equation for constraint g=c. (3) Solve g for a variable and substitute into f. (4) find critical points of the function created in step 3. (5) classify critical points.
- Critical points (gradient = zero vector, or gradient is undefined)
- Second derivative test for functions of two variables (does <u>not</u> work where gradient is undefined!)

15.2 Global Extrema on an unbounded domain
- Thm: if f is a 2^{nd} degree polynomial, then a local extremum is also a global extremum.
- Application problems: Linear regression, etc…

15.3 : Using Lagrange Multipliers to find extrema of f on the curve: g=c or on a bounded domain: g<=c

- Thm: if f is continuous on a closed and bounded region R, then f must have global extrema
- Extrema of f subject to the constraint g=c can occur

  1) at points satisfying the system of equations: grad f = λ(grad g) AND g=c
  2) at end points or corners of constraint curve, g=c
  (note: you <u>cannot</u> use 2^{nd} derivatives test to classify these points)
- Extrema of f subject to the constraint g<=c can occur
  1) at points satisfying the system of equations: grad f = λ(grad g) AND g=c
  2) or at end points or corners of constraint curve, g=c
  3) critical points of f that also satisfy g<=c